Sensitivity of Redshift Distortion Measurements to Cosmological Parameters

Andrew A. de Laix and Glenn Starkman

Physics Department, Case Western Reserve University, Cleveland, OH 44106-7079

ABSTRACT

The multipole moments of the power spectrum of large scale structure, observed in redshift space, are calculated for a finite sample volume including the effects of both the linear velocity field and geometry. A variance calculation is also performed including the effects of shot noise. The sensitivity with which a survey with the depth and geometry of the Sloan Digital Sky Survey (SDSS) can measure cosmological parameters Ω_0 and b_0 (the bias) or λ_0 (the cosmological constant) and b_0 is derived through fitting power spectrum moments to the large scale structure in the linear regime in a way which is independent of the evolution of the galaxy number density. A fiducial model is assumed and the region of parameter space which can then be excluded to a given confidence limit is determined. In the absence of geometric and evolutionary effects, the ratios of multipole moments (in particular the zeroth and second), are degenerate for models of constant $\beta \approx \Omega^{0.6}/b_0$. However, this degeneracy is broken by the Hubble expansion, so that in principle Ω_0 and b_0 may be measured separately by a deep enough galaxy redshift survey (Nakamura, Matsubara, & Suto (1997)). We find that for surveys of the approximate depth of the SDSS no restrictions can be placed on Ω_0 at the 99% confidence limit when a fiducial open, $\Omega_0 = 0.3$ model is assumed and bias is unconstrained. At the 95% limit, $\Omega_0 < .85$ is ruled out. Furthermore, for this fiducial model, both flat (cosmological constant) and open models are expected to reasonably fit the data. For flat, cosmological constant models with a fiducial $\Omega_0 = 0.3$, we find that models with $\Omega_0 > 0.48$ are ruled out at the 95% confidence limit regardless of the choice of the bias parameter, and open models cannot fit the data even at the 99% confidence limit. We also find significant deviations in β from the naive estimate for both fiducial models. Thus, we conclude for the SDSS that linear evolution-free statistics alone can strongly distinguish between $\Omega_0 = 1$ and low matter density models only in the case of the fiducial cosmological constant model. For the open model, $\Omega_0 = 1$ is only at best only nominally excluded unless $\Omega_0 < 0.3$.

Subject headings: cosmology: Large Scale Structure of the Universe

1. Introduction

With the expectation of new and much larger samples of galaxies with measured redshifts, *e.g.* the Sloan Digital Sky Survey (SDSS, see *e.g.* Gunn & Weinberg (1995), Strauss (1997)), it is opportune to reconsider what information may be extracted from these data. In particular, we would like to determine how well fundamental cosmological parameters like the current matter density Ω_0 and cosmological constant λ_0 can be inferred from redshift surveys. Perhaps the simplest approach would be to just measure the mean number density of galaxies as a function of redshift. Unfortunately, this number density is evolving through processes other than cosmological expansion, and these factors must be accurately deconvolved if we are to extract the desired information about cosmology. Alternatively, in the standard cosmological model, the growth of perturbations in galaxy number density is driven only by gravity, so we expect that measurements derived from these variations will be cleaner signals of cosmology. Locally, this perturbation growth induces peculiar-velocities in the galaxies which lead to distortions in redshift maps. At increasing redshift, the effects of these distortions evolve due to the changing growth rate of perturbations, and due to the geometry and expansion of the universe. This evolution, in turn, depends on fundamental cosmological parameters like the curvature, Ω_0, and λ_0, so one may hope to infer values of these parameters by observing the change in the distortions with redshift. Nakamura, Matsubara, & Suto (1997) have recently calculated the effects on the expectation value of the various multipoles of the correlation function, to first order in the redshift. However, they made no estimate of the expected errors, necessary for determining the sensitivity of any real survey.

In this paper we shall derive the mean, cosmic variance and shot noise associated with the statistics of multipole moments of the linear power spectrum calculating the redshift-dependent effects to all orders. Note that moments beyond the zeroth are induced by redshift distortions and would not be present in the mean for a real-space survey, *i.e.* a survey which plots galaxies at their conformal distances. Then, we estimate the sensitivity of these statistics in the linear regime for a survey with depth and geometry comparable to the SDSS by determining those models which can be excluded to given confidence level when a particular fiducial model is assumed. In this way, we hope to put into perspective the ability of upcoming redshift surveys to fix cosmological parameters in the absence of external constraints.

2. Redshift Distortions in The Power Spectrum

The theory of the linear distortions of redshift surveys with regards to the underlying real–space distribution was first investigated by Kaiser (1987) for shallow surveys where redshift evolution and geometrical effects were negligible. This work has been extended to include both evolution and geometry for larger redshifts by Matsubara & Suto (1996) and Ballinger, Peacock, & Heavens (1996); for convenience and clarity, we shall repeat their work following the formalism of Matsubara and Suto. We begin with the standard assumption of a Friedman-Robertson-Walker universe with a metric given by

$$ds^2 = -dt^2 + a(t)^2\{d\chi^2 + S(\chi)^2(d\theta^2 + \sin^2\theta\ d\phi^2)\}, \tag{1}$$

where $S(\chi)$ is determined by the geometry of the universe through the spatial curvature K, and

$$S(\chi) = \begin{cases} \sin(\sqrt{K}\chi)/\sqrt{K} & (K > 0), \\ \chi & (K = 0), \\ \sinh(\sqrt{-K}\chi)/\sqrt{-K} & (K < 0). \end{cases} \tag{2}$$

The curvature is given in terms of the present day Hubble constant H_0, the matter energy density Ω_0 and the energy density in cosmological constant λ_0:

$$K = H_0^2(\Omega_0 + \lambda_0 - 1), \tag{3}$$

where we assume that the present scale factor a_0 is unity. The proper radial distance from an observer to a source, χ, can be determined from the integral

$$\chi = \int_t^{t_0} \frac{dt}{a(t)} = \int_0^z \frac{dz}{H(z)}, \tag{4}$$

where in the final integral, we introduce the redshift–dependent Hubble parameter

$$H(z) = H_0\sqrt{\Omega_0(1+z)^3 + (1 - \Omega_0 - \lambda_0)(1+z)^2 + \lambda_0}. \tag{5}$$

To see how redshift-space maps are distorted with regards to their real–space counterparts, we need to consider both the geometry of the universe and the peculiar–velocities of the objects being observed. Let us first consider geometry. We would like to examine small displacements about a given origin located at a redshift z with respect to a terrestrial observer ($z = 0$). This small displacement is represented by the vector $\vec{x}$ in comoving, real–space coordinates and has components $x_\parallel$ parallel to the line–of–sight and $x_\perp$ perpendicular to the line–of–sight. To first order in the Taylor expansion we may write

$$\begin{aligned} x_\parallel &= \frac{d\chi(z)}{dz}\delta z = \frac{c_\parallel}{H_0}\delta z \\ \vec{x}_\perp &= S(\chi(z))\delta\vec{\theta} = \frac{c_\perp z}{H_0}\delta\vec{\theta}, \end{aligned} \tag{6}$$

where $c_\parallel = H_0/H(z)$ and $c_\perp = H_0 S(\chi(z))/z$. We have also assumed a distant observer by linearizing in $\delta\theta$. Since $c_\parallel$ and $c_\perp$ have different functional forms, a spherical object will be distorted into an ellipse when mapped into redshift space, and it is this effect that Alcock & Paczyński (1979) suggested could be used to measure cosmological parameters.

For galaxy surveys we also need to consider how the peculiar velocities of objects affect redshift distributions. A galaxy that would be observed at redshift z_{true} in the absence of any peculiar motion, and which has a non-relativistic peculiar velocity $\vec{v}$ relative to the background, is seen by an observer with non-relativistic peculiar velocity $\vec{v}^0$ with an apparent redshift

$$1 + z_{app} = (1 + z_{true})(1 + v_\parallel - v_\parallel^0). \tag{7}$$

Here $v_\parallel$ is the peculiar velocity of the object projected along the line–of–sight. Using eq. (7) along with the eq. (6), we can calculate the displacement in redshift-space $\vec{s}$ for a given $\vec{x}$ including both the geometric and velocity effects. If we drop the term proportional to $Hx_\parallel(1 + v_\parallel - v_\parallel^O)$, which is small in comparison to the others, we can easily show that

$$
\begin{aligned}
s_1 &= \frac{x_1}{c_\perp(z)}, \ s_2 = \frac{x_2}{c_\perp(z)}, \\
s_3 &= \frac{z_{app} - z}{H_0} \simeq \frac{1}{c_\parallel(z)}\left[x_3 + \frac{1+z}{H(z)}(v_\parallel - v_\parallel^O)\right],
\end{aligned} \tag{8}
$$

where we recall that z is the redshift of the origin of reference and $z_{true} - z$ is equivalent to δz in eq. (6). Note that the 3 direction points along the line–of–sight. The above expressions now give us almost everything we need to know to relate the real–space density–contrast to the redshift-space contrast. The conversion is made by considering the Jacobian transformation from $\vec{x}$ to $\vec{s}$:

$$\left|\frac{\partial\vec{x}}{\partial\vec{s}}\right| \approx c_\perp^2 c_\parallel \left[1 - \frac{1+z}{H(z)}\frac{\partial}{\partial x_3}v_\parallel(\vec{x})\right], \tag{9}$$

to linear order in the velocity perturbation, where the local density in real space $\delta^r(vecx)$ is thus enhanced by this factor when observed in redshift space. For the mean density, we take the ensemble average, so the velocity term vanishes when averaging the Jacobian. Putting everything together, we get

$$\delta^s(\vec{s}(\vec{x})) = \delta^r(\vec{x}) - \frac{1+z}{H(z)}\frac{\partial}{\partial x_3}v_\parallel(\vec{x}). \tag{10}$$

This result is true for any fluid component, e.g. galaxies, dark matter, etc., as it depends only on the continuity of the fluid, but in the special case of the *total mass* fluctuations δ^m we can go further and replace the velocity term by assuming, in the linear regime, that fluctuations grow only by gravitational

instability. Perturbation theory gives us a relationship between δ^m and the peculiar velocity, namely

$$v_{\parallel}(\vec{x}) = -\frac{H(z)}{1+z}f(z)\partial_3\triangle^{-1}\delta^m(\vec{x}),\tag{11}$$

where $\triangle^{-1}$ is the inverse of the Laplacian operator (see *e.g.* Peebles 1980) and $f(z)$ is the logarithmic derivative of the linear growth rate $D(z)$,

$$f(z) = \frac{d\ln D(z)}{d\ln a} \simeq \Omega(z)^{0.6} + \frac{\lambda(z)}{70}\left[1+\frac{\Omega(z)}{2}\right].\tag{12}$$

In this equation, the redshift dependent cosmological parameters $\Omega(z)$ and $\lambda(z)$ are given by

$$\Omega(z) = \left[\frac{H_0}{H(z)}\right]^2(1+z)^3\Omega_0,\ \lambda(z) = \left[\frac{H_0}{H(z)}\right]^2\lambda_0,\tag{13}$$

(during the matter-dominated epoch) while the linear growth factor has the well known solution (Peebles (1980))

$$D(z) = \frac{5\Omega_0 H_0^2}{2}H(z)\int_z^\infty \frac{1+z'}{H(z')^3}dz'.\tag{14}$$

Galaxies do not necessarily trace mass, but, to first order we may assume that the relationship between galaxy and mass fluctuations is linear, given by a bias factor $\delta^r = b(z)\delta^m$. The redshift dependence of this bias factor can be quite complex, depending on the dynamics of galaxy formation, but for $z \ll 1$ it is reasonable to follow Fry (1996) and suppose that all the galaxies in our survey were formed well prior to their observation with some intrinsic bias. Treating these galaxies as a separate matter component in the perturbation equations, Fry has shown that

$$b(z) = 1 + \frac{D(0)}{D(z)}(b_0 - 1).\tag{15}$$

Putting everything together, we may now write the linear overdensity function as a function of $\vec{s}$ in the neighborhood of a given origin at z:

$$\delta^s(\vec{s}(\vec{x})) = \delta^r(\vec{x}) + \beta(z)\frac{\partial^2}{\partial x_3^2}\triangle^{-1}\delta^r(\vec{x}),\tag{16}$$

where $\beta(z) = f(z)/b(z)$. Both for expressing the density fluctuations, usually assumed to be a Gaussian random field, and for evaluating the inverse Laplacian, it is convenient to work in Fourier rather than real space. Thus we can write

$$\delta^s(\vec{s}(\vec{x})) = \int\frac{d^3k}{(2\pi)^3}\left[1+\beta(z)\frac{k_3^2}{k^2}\right]\frac{b(z)D(z)}{b(0)D(0)}\tilde{\delta}^r(\vec{k})e^{i\vec{k}\cdot\vec{x}(\vec{s})},\tag{17}$$

where $\tilde{\delta}(\vec{k})$ is the Fourier transform of the real–space density contrast evaluated at $z = 0$.

The above result is more than adequate for perturbations on the largest scales, but on smaller scales, we must account for the non–linear evolution. This causes the density fluctuations to rapidly virialize into halos, and the effect on the redshift–space density field is well approximated by adding a random velocity to all the mass elements. Empirical evidence suggests that the distribution of these velocities is well described by a power–law model with dispersion σ_v (Cole, Fisher, & Weinberg (1995)). Thus, the corrected density fluctuation in redshift space then takes the form

$$\delta^s(\vec{s}(\vec{x})) = \int \frac{d^3k}{(2\pi)^3} \left[1 + \beta(z)\frac{k_3^2}{k^2}\right] \left[1 + \frac{k_3^2\sigma_v^2}{2}\right]^{-1} \frac{b(z)D(z)}{b(0)D(0)} \tilde{\delta}^r(\vec{k}) e^{i\vec{k}\cdot\vec{x}(\vec{s})}. \tag{18}$$

Now let us consider the Fourier coefficients $\tilde{\delta}^s(\vec{K})$, which we would derive from the redshift-space density distribution, defined as

$$\tilde{\delta}^s(\vec{K}) \equiv \int d^3s \; \delta^s(\vec{s}(\vec{x})) e^{-i\vec{K}\cdot\vec{s}}. \tag{19}$$

Substituting in the definition of $\delta^s(\vec{s}(\vec{x}))$ from eq. (18), we get

$$\tilde{\delta}^s(\vec{K}) = \int d^3s \int \frac{d^3k}{(2\pi)^3} \left[1 + \beta(z)\frac{k_3^2}{k^2}\right] \left[1 + \frac{k_3^2\sigma_v^2}{2}\right]^{-1} \frac{b(z)D(z)}{b(0)D(0)} \tilde{\delta}^r(\vec{k}) e^{i\vec{k}\cdot\vec{x}(\vec{s})} e^{-i\vec{K}\cdot\vec{s}}. \tag{20}$$

Recall that $\vec{x}(\vec{s}) = c_\perp \vec{s}_\perp + c_\parallel \vec{s}_\parallel$, so by performing the integration over d^3s we are left with

$$\tilde{\delta}^s(\vec{K}) = \int d^3k \left[1 + \beta(z)\frac{k_3^2}{k^2}\right] \left[1 + \frac{k_3^2\sigma_v^2}{2}\right]^{-1} \frac{b(z)D(z)}{b(0)D(0)} \tilde{\delta}^r(\vec{k}) \delta^D(\vec{K} - c_\perp \vec{k}_\perp + c_\parallel \vec{k}_\parallel), \tag{21}$$

where $\delta^D(\vec{K} - c_\perp \vec{k}_\perp + c_\parallel \vec{k}_\parallel)$ is a Dirac delta function. The final integration is now trivial to perform, and doing so leaves us with

$$\begin{aligned}
\tilde{\delta}^s(\vec{K}) &= \left[1 + \frac{\beta\mu^2 c_\parallel^{-2}}{\left(c_\parallel^{-2} - c_\perp^{-2}\right)\mu^2 + c_\perp^{-2}}\right] \left(1 + \frac{1}{2}c_\parallel^{-2}\sigma_v^2 K^2 \mu^2\right)^{-1} \\
&\quad \times \tilde{\delta}^r\left(K\sqrt{(c_\parallel^{-2} - c_\perp^{-2})\mu^2 + c_\perp^{-2}}\right)(c_\parallel c_\perp^2),
\end{aligned} \tag{22}$$

where $\mu = K_3/K$.

3. Fluctuations in Galaxy Counts

In the previous section, we derived an expression for the Fourier components of the galaxy density field mapped into redshift space valid in the neighborhood of some observation point. Unfortunately, the results fail as $\vec{s}$ grows large enough to break the linearization constraints enforced to calculate $c_\parallel$ and $c_\perp$, so we can only apply them to small volumes. The question becomes, how can we best use these results to

analyze redshift surveys which cover large angles of sky and are deep in redshift space? The only practical alternative is to break up such a survey into many sub–volumes, and measure the distortions within each of them. These measurements can then be fit to theoretical calculations and used to determine cosmological parameters. In §3.1, we shall discuss the calculation of multipole moments of the power spectrum as observed in a single sub–volume, while in §3.2 we will calculate the cosmic variance of these moments, for a single volume. Finally, in §3.3, we calculate the shot noise or finite sampling variance for this sub–volume. Together, these computations will allow us to determine the sensitivity of redshift surveys which contain many statistically independent sub–volumes.

3.1. Moments of the Galaxy Distribution

The mean value for multipole moments of the power spectrum derived from a finite sample window were first calculated in Cole, Fisher, & Weinberg (1994); here we repeat their calculation, to which we will add a calculation of the variance of these statistics in the following subsections. Since we are considering galaxy surveys as our source of data, we must consider not continuous density fields, but discrete realizations of those fields given by the point locations of the galaxies. This is a well understood problem, so we shall follow Peebles (1980), and subdivide our survey into a large number of cells such that the probability of finding two galaxies in a cell is vanishingly small in comparison to that of finding one in a cell. We define N_i to be the number of galaxies in cell i, where by our supposition N_i is either one or zero. In the previous section, we considered the Fourier components of the density fluctuations in a neighborhood about a fixed redshift. For a corresponding survey, we ought to only consider galaxies inside a window such that the linearization performed previously is a valid approximation, and thus we will weight each of our cells by a window function $w(\vec{s}_i)$ with volume V_w. Also, real galaxy surveys usually do not uniformly sample all the volume in the survey, *e.g.* a flux limited survey sees fewer and fewer objects as distance increases; the function $\phi(\vec{s}_i)$ will represent this selection effect. Putting everything together, we see that a sensible way to calculate the Fourier coefficients of the density fluctuations is

$$\tilde{\delta}_{\vec{K}} = \frac{1}{nV_w} \sum_i \left(N_i - \langle N_i \rangle \right) \frac{w(\vec{s}_i)}{\phi(\vec{s}_i)} e^{-i\vec{K}\cdot\vec{s}_i}, \tag{23}$$

where n is the mean number density in the volume and angle brackets refer to the ensemble average. Dividing by ϕ will, as we shall see, cancel its effects on the statistics we calculate rendering selection-independent results. The expectation–value for the number of galaxies in cell i is given by $\langle N_i \rangle = n\phi(\vec{s}_i)d^3 s_i$. As we shall see momentarily, dividing the $\tilde{\delta}_{\vec{K}}$'s by the selection function will remove the effects of the latter from

the statistics we calculate.

In the previous section we showed that the Fourier coefficients of the density field in redshift space are dependent on μ, the cosine of the angle formed by $\vec{K}$ and the observation axis. Furthermore, the angular dependence is a function of cosmology which appears through β, $c_\parallel$ and $c_\perp$. One might hope Nakamura, Matsubara, & Suto (1997) that by measuring the angular dependence of the $\delta_{\vec{K}}$'s one can determine the underlying cosmology, and thus, to that end, we consider the multipole moments of the square of the Fourier coefficients

$$\bar{p}_\ell(K) \equiv \frac{2\ell+1}{4\pi} \int d\Omega_K \mathcal{P}_\ell(\mu_K) \left\langle \tilde{\delta}(\vec{K})\tilde{\delta}(\vec{K})^* \right\rangle, \tag{24}$$

where $\mathcal{P}_\ell$ is a Legendre polynomial. We choose the square of the Fourier coefficient because the ensemble average of the coefficient itself is zero and therefore we would see no signal in the mean. Substituting in for $\tilde{\delta}_{\vec{K}}$ from eq. (23) and taking the ensemble average, we find

$$\bar{p}_\ell(K) = \frac{2\ell+1}{4\pi} \int d\Omega_K \mathcal{P}_\ell(\mu_K) \frac{1}{n^2 V_w^2} \sum_{i,j} \langle (N_i - \langle N_i \rangle)(N_j - \langle N_j \rangle) \rangle \frac{w(\vec{s}_i)w(\vec{s}_j)}{\phi(\vec{s}_i)\phi(\vec{s}_j)} e^{i\vec{K}\cdot(\vec{s}_i - \vec{s}_j)} \tag{25}$$

The remaining expectation–value can be written

$$\langle (N_i - \langle N_i \rangle)(N_j - \langle N_j \rangle) \rangle = n^2 \phi(\vec{s}_i)\phi(\vec{s}_j)d^3 s_i d^3 s_j \xi_{ij} + n\phi(\vec{s}_i)d^3 s_i \delta_{ij}, \tag{26}$$

where δ_{ij} is the Kroniker delta, ξ_{ij} is the correlation function given by

$$\xi_{ij} \equiv \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{s}_i - \vec{s}_j)} P(\vec{k}), \tag{27}$$

and $P(\vec{k}) = \tilde{\delta}^s(\vec{k})\tilde{\delta}^{s*}(\vec{k})/(2\pi)^3$ is the redshift–space power–spectrum derived from the perturbation spectrum given in eq. (22). It may be conveniently expressed as an expansion in multipoles

$$P(\vec{k}) = \sum_\ell \mathcal{P}_\ell(\mu_k) P_\ell(k), \tag{28}$$

where $\mu_k = k_3/k$ is the cosine of the angel formed between $\vec{k}$ and the line of sight. Putting everything together, we find

$$\begin{aligned} \bar{p}_\ell(K) &= \frac{2\ell+1}{4\pi} \int d\Omega_K \mathcal{P}_\ell(\mu_K) \frac{1}{n^2 V_w^2} \sum_{i,j} \left[n^2 \phi(\vec{s}_i)\phi(\vec{s}_j)d^3 s_i d^3 s_j \right. \\ &\quad \left. \times \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{s}_i - \vec{s}_j)} P(\vec{k}) + n\phi(\vec{s}_i)d^3 s_i \delta_{ij} \right] \frac{w(\vec{s}_i)w(\vec{s}_j)}{\phi(\vec{s}_i)\phi(\vec{s}_j)} e^{i\vec{K}\cdot(\vec{s}_i - \vec{s}_j)}. \end{aligned} \tag{29}$$

In the limit of infinitesimal volumes, the sums over i and j reduce to integrals, and we see in the first term that these spatial integrals give us Fourier transforms, reducing the above equation to

$$\bar{p}_\ell(K) = \frac{2\ell+1}{4\pi} \int d\Omega_K \mathcal{P}_\ell(\mu_K) \int \frac{d^3 k}{(2\pi)^3} P(\vec{k}) \left| \tilde{w}^{(1)}(\vec{K} + \vec{k}) \right|^2 + \frac{\delta_{\ell 0}}{n V_w^2} \int d^3 s \frac{|w(\vec{s})|^2}{\phi(\vec{s})}, \tag{30}$$

where

$$\tilde{w}^{(p)}(\vec{k}) \equiv \int \frac{d^3s}{V_w} w^p(\vec{s}) e^{i\vec{k}\cdot\vec{s}}. \tag{31}$$

To further reduce the integral in eq. (30), we shall assume that our window function is spherically symmetric implying $\tilde{w}(\vec{k}) = \tilde{w}^*(\vec{k}) = \tilde{w}(|\vec{k}|)$. We can then write out the multipole expansion

$$\left|\tilde{w}^{(1)}(\vec{K}+\vec{k})\right|^2 = \left[\tilde{w}^{(1)}(\vec{K}+\vec{k})\right]^2 \equiv \tilde{W}^{(1,1)}(\vec{K}+\vec{k}) \equiv \sum \mathcal{P}_n(\mu_{K,k})\tilde{W}_n^{(1,1)}(K,k), \tag{32}$$

where $\mu_{K,k}$ is the cosine of the angle formed between $\vec{K}$ and $\vec{k}$. More generally, we will define

$$\tilde{W}^{(p,q)}(\vec{K}+\vec{k}) \equiv \tilde{w}^{(p)}(\vec{K}+\vec{k})\tilde{w}^{(q)}(\vec{K}+\vec{k}) \tag{33}$$

and use the multipole expansions

$$\tilde{w}^{(p)}(\vec{K}+\vec{k}) \equiv \sum \mathcal{P}_n(\mu_{K,k})\tilde{w}_n^{(p)}(K,k). \tag{34}$$

and

$$\tilde{W}^{(p,q)}(\vec{K}+\vec{k}) \equiv \sum \mathcal{P}_n(\mu_{K,k})\tilde{W}_n^{(p,q)}(K,k). \tag{35}$$

As we have already mentioned (eq. (28)), $P(\vec{k})$ can also be written as a multipole expansion in μ_k. To make use of these expansions, we will apply a well known property of Legendre polynomials, namely

$$\int d\Omega_k \mathcal{P}_n(\mu_{K,k})\mathcal{P}_m(\mu_k) = \frac{4\pi}{2n+1}\mathcal{P}(\mu_K)\delta_{nm}. \tag{36}$$

Using this theorem along with the expansions of the window function and power spectrum, one can see after some algebra that

$$\bar{p}_\ell(K) = \frac{4\pi}{2\ell+1}\int_0^\infty \frac{k^2 dk}{(2\pi)^3} P_\ell(k)\tilde{W}_\ell^{(1,1)}(K,k) + \frac{\delta_{\ell 0}}{nV_w^2}\int d^3s \frac{|w(\vec{s})|^2}{\phi(\vec{s})}, \tag{37}$$

where the second term vanishes as the number of galaxies in the survey volume grows large. Conversely, it is easily removed from measurements in surveys for which shot noise makes a significant contribution.

3.2. Variance of the Moments

Now that we have derived the multipole moments of the power spectrum for a finite volume, one can imagine taking many such volumes in a redshift survey and calculating $\bar{p}_\ell$'s as a function of z. Then by using the theoretical results, one could fit various cosmological models to the data and determine the best fit. Our ability to fit the data is limited by two types of noise: shot noise and cosmic variance. Shot noise

arises because galaxy surveys do not include an infinite number of objects, but it scales as the inverse of the number of galaxies in the sample and is negligible when $N \gg 1$. The second source, cosmic variance, arises from the simple fact that we live in only a particular realization of the ensemble of possible universes. No amount of galaxy sampling can eliminate this noise, so it sets a theoretical upper limit to the accuracy of any measurements that we might make. In this section we will calculate the intrinsic cosmic variance that we can expect in our measurements of the moments of the power spectrum for a single window, representing the ideal limit achievable by any redshift survey.

Let us begin with the usual definition for the covariance, namely $\sigma^2_{\ell,\ell'}(K, K') = \langle \bar{p}_{\ell'}(K') \bar{p}_\ell(K) \rangle - \langle \bar{p}_\ell(K) \rangle \langle \bar{p}_{\ell'}(K') \rangle$. We choose different magnitudes for K and K' so as to treat the most general case. Using our definition of $\bar{p}_\ell$ we can write

$$
\begin{aligned}
\sigma^2_{\ell,\ell'}(K, K') &= \left[\frac{2\ell + 1}{4\pi} \frac{2\ell' + 1}{4\pi} \int d\Omega_K d\Omega_{K'} \mathcal{P}(\mu_K) \mathcal{P}(\mu_{K'}) \frac{1}{n^4 V_w^4} \right. \\
&\quad \times \sum_{i,j,k,l} \frac{w(\vec{s}_i) w(\vec{s}_j) w(\vec{s}_k) w(\vec{s}_l)}{\phi(\vec{s}_i) \phi(\vec{s}_j) \phi(\vec{s}_k) \phi(\vec{s}_l)} e^{i\vec{K} \cdot (\vec{s}_i - \vec{s}_j)} e^{i\vec{K}' \cdot (\vec{s}_k - \vec{s}_l)} \\
&\quad \left. \times \langle (N_i - \langle N_i \rangle)(N_j - \langle N_j \rangle)(N_k - \langle N_k \rangle)(N_l - \langle N_l \rangle) \rangle \right] \\
&\quad - \langle \bar{p}_\ell(K) \rangle \langle \bar{p}_{\ell'}(K') \rangle.
\end{aligned}
\tag{38}
$$

In the limit that nV_w grows large, *i.e* when ignoring shot noise, it can easily be shown that the ensemble average of the product of N's is

$$
n^4 d^3 s_i \ d^3 s_j \ d^3 s_k \ d^3 s_l \ \phi(\vec{s}_i) \phi(\vec{s}_j) \phi(\vec{s}_k) \phi(\vec{s}_l) \left[\xi_{ij} \xi_{kl} + \xi_{ik} \xi_{jl} + \xi_{il} \xi_{kj} \right],
\tag{39}
$$

if the underlying distribution is Gaussian. Restricting ourselves to Gaussian fields is reasonable on large scales, particularly for inflationary models of structure formation. Putting things together, we get

$$
\begin{aligned}
\sigma^2_{\ell,\ell'} &= \left[\frac{2\ell + 1}{4\pi} \frac{2\ell' + 1}{4\pi} \int d\Omega_K d\Omega_{K'} \mathcal{P}(\mu_K) \mathcal{P}(\mu_{K'}) \frac{1}{V_w^4} \right. \\
&\quad \times \sum_{i,j,k,l} w(\vec{s}_i) w(\vec{s}_j) w(\vec{s}_k) w(\vec{s}_l) e^{i\vec{K} \cdot (\vec{s}_i - \vec{s}_j)} e^{i\vec{K} \cdot (\vec{s}_k - \vec{s}_l)} \\
&\quad \left. \times \left[\xi_{ij} \xi_{kl} + \xi_{ik} \xi_{jl} + \xi_{il} \xi_{kj} \right] d^3 s_i d^3 s_j d^3 s_k d^3 s_l \right] \\
&\quad - \langle \bar{p}_\ell(K) \rangle \langle \bar{p}_{\ell'}(K') \rangle
\end{aligned}
\tag{40}
$$

The three combinations of two-point correlation functions $\xi_{ij} \xi_{kl}$, $\xi_{ik} \xi_{jl}$, $\xi_{il} \xi_{kj}$, give us three different terms to calculate, which we call I_1, I_2, and I_3 respectively. Close inspection of I_1 reveals that it is exactly equivalent to $\langle \bar{p}_\ell(K) \rangle \langle \bar{p}_{\ell'}(K') \rangle$ canceling the final term. Thus we are left to calculate I_2 and I_3.

Now let us take a closer look at I_2. As before, we can do the spatial integrals and replace the window functions with their Fourier transforms. The resulting equation is

$$
\begin{aligned}
I_2 &= \frac{2\ell+1}{4\pi}\frac{2\ell'+1}{4\pi}\int d\Omega_K d\Omega_{K'}\mathcal{P}(\mu_K)\mathcal{P}(\mu_{K'})\int\frac{d^3k}{(2\pi^3)}\frac{d^3k'}{(2\pi^3)}P(\vec{k})P(\vec{k'}) \\
&\quad\times\tilde{w}^{(1)}(\vec{K}+\vec{k})\tilde{w}^{(1)}(\vec{K}-\vec{k'})\tilde{w}^{(1)}(\vec{K'}+\vec{k'})\tilde{w}^{(1)}(\vec{K'}-\vec{k}).
\end{aligned}
\tag{41}
$$

We now expand the window function in a multipole series:

$$
\begin{aligned}
\tilde{w}^{(1)}(\vec{K}+\vec{k}) &\equiv \sum_n \tilde{w}_n^{(1)}(k,K)\mathcal{P}_n(\mu_{k,K}) \\
&= \sum_n\sum_{m=-n}^{n}\tilde{w}_n^{(1)}(k,K)\frac{4\pi}{2n+1}Y_{n,m}(\Omega_K)Y_{n,m}^*(\Omega_k),
\end{aligned}
$$

where Y_{nm} is the usual spherical harmonic and the last line was derived from the addition theorem of spherical harmonics. Expanding similarly the other window functions, we may rewrite eq. (41) as

$$
\begin{aligned}
I_2 &= \sum_{n_1,m_1}\sum_{n_2,m_2}\sum_{n_3,m_3}\sum_{n_4,m_4}\frac{(4\pi)^2(2\ell+1)(2\ell'+1)}{(2n_1+1)(2n_2+1)(2n_3+1)(2n_4+1)} \\
&\quad\times\int\frac{d^3k}{(2\pi)^3}P(\vec{k})(-1)^{n_3}\tilde{w}_{n_1}^{(1)}(k,K)\tilde{w}_{n_3}^{(1)}(k,K')Y_{n_1,m_1}^*(\Omega_k)Y_{n_3,m_3}^*(\Omega_k) \\
&\quad\times\int\frac{d^3k'}{(2\pi)^3}P(\vec{k'})(-1)^{n_2}\tilde{w}_{n_2}^{(1)}(k',K)\tilde{w}_{n_4}^{(1)}(k',K')Y_{n_2,m_2}^*(\Omega_{k'})Y_{n_4,m_4}^*(\Omega_{k'}) \\
&\quad\times\int d\Omega_K\mathcal{P}_\ell(\mu_K)Y_{n_1,m_1}(\Omega_K)Y_{n_2,m_2}(\Omega_K) \\
&\quad\times\int d\Omega_{K'}\mathcal{P}_{\ell'}(\mu_{K'})Y_{n_3,m_3}(\Omega_{K'})Y_{n_4,m_4}(\Omega_{K'}).
\end{aligned}
\tag{42}
$$

Note that the factors $(-1)^{n_2}$ and $(-1)^{n_3}$ arise from the fact that $\mathcal{P}_n(-x)=(-1)^n\mathcal{P}_n(x)$. From the above equation, we see two integrals of the form

$$
\int d\Omega_K\mathcal{P}_\ell(\mu_K)Y_{n_1,m_1}(\Omega_K)Y_{n_2,m_2}(\Omega_K),
\tag{43}
$$

which evaluate to

$$
\sqrt{\frac{4\pi}{2\ell+1}}C_{n_1,\ell,n_2;m_1}\delta_{m_2,-m_1},
\tag{44}
$$

where $C_{n_1,\ell,n_2;m_1}\delta_{m_2,-m_1}=0$ unless the triangle condition $|n_1-\ell|\le n_1\le n_1+\ell$ is satisfied, in which case

$$
C_{n_1,\ell,n_2;m_1}\delta_{m_2,-m_1} = \sqrt{\frac{(2\ell+1)(2n_1+1)(2n_2+1)}{4\pi}}\begin{pmatrix} n_1 & \ell & n_2 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} n_1 & \ell & n_2 \\ m_1 & 0 & -m_1 \end{pmatrix}.
\tag{45}
$$

The arrays in parentheses are the Wigner $3j$ symbols. For a complete discussion of these symbols see, for example, Edwards (1957). Also, a fast recursive algorithm for calculating the $3j$ symbols is described in Schulten & Gordon (1975).

The second type of integral which appears in eq. (42) has the form

$$\int \frac{k^2 dk}{(2\pi)^3} \tilde{w}_{n_1}^{(1)}(k, K) \tilde{w}_{n_3}^{(1)}(k, K') \int d\Omega_k P(\vec{k}) Y_{n_1, m_1}^*(\Omega_k) Y_{n_3, m_3}^*(\Omega_k). \tag{46}$$

Expanding the power spectrum into its multipole moments, we can write this integral as

$$\sum_i \mathcal{S}_{n_1, n_3; i}(K, K') \sqrt{\frac{4\pi}{2i+1}} C_{n_1, i, n_3; -m_1} \delta_{m_3, -m_1}, \tag{47}$$

where we have defined the function

$$\mathcal{S}_{n_1, n_3; i}(K, K') \equiv \int \frac{k^2 dk}{(2\pi)^3} \tilde{w}_{n_1}^{(1)}(k, K) \tilde{w}_{n_3}^{(1)}(k, K') P_i(k). \tag{48}$$

Now we can combine all of the terms together to write

$$\begin{aligned} I_2 &= \sum_{i,j} \sum_{n_1, m_1} \cdots \sum_{n_4, m_4} \delta_{m_1, -m_2} \delta_{m_1, -m_3} \delta_{m_4, -m_2} \delta_{m_4, -m_3} (-1)^{n_2 + n_3} \\ &\times \frac{(4\pi)^4 \, \mathcal{S}_{n_1, n_3; i}(K, K') \mathcal{S}_{n_2, n_4; j}(K, K') \sqrt{2\ell+1} \sqrt{2\ell'+1}}{(2n_1+1)(2n_2+1)(2n_3+1)(2n_4+1) \sqrt{2i+1} \sqrt{2j+1}} \\ &\times C_{n_1, i, n_3; -m_1} C_{n_2, j, n_4; -m_1} C_{n_1, \ell, n_2; m_1} C_{n_3, \ell', n_4; m_3}, \end{aligned} \tag{49}$$

with the implicit constraints

$$\begin{aligned} |n_1 - \ell| &\leq n_2 \leq n_1 + \ell, \\ |n_1 - n_3| &\leq i \leq n_1 + n_3, \\ |n_3 - \ell'| &\leq n_4 \leq n_3 + \ell', \\ |n_2 - n_4| &\leq j \leq n_2 + n_4. \end{aligned}$$

Applying all of the constraints and summing out the Kroniker delta's, I_2 takes the final form

$$\begin{aligned} I_2 &= \sum_{n_1, n_3} \sum_{n_2 = |n_1 - \ell|}^{n_1 + \ell} \sum_{n_4 = |n_3 - \ell'|}^{n_3 + \ell'} \sum_{i = |n_1 - n_3|}^{n_1 + n_3} \sum_{j = |n_2 - n_4|}^{n_2 + n_4} \sum_{m_1 = -n_1}^{n_1} \\ &\times (-1)^{n_2 + n_3} \frac{(4\pi)^4 \, \mathcal{S}_{n_1, n_3; i}(K, K') \mathcal{S}_{n_2, n_4; j}(K, K') \sqrt{2\ell+1} \sqrt{2\ell'+1}}{(2n_1+1)(2n_2+1)(2n_3+1)(2n_4+1) \sqrt{2i+1} \sqrt{2j+1}} \\ &\times C_{n_1, i, n_3; -m_1} C_{n_2, j, n_4; -m_1} C_{n_1, \ell, n_2; m_1} C_{n_3, \ell', n_4; m_1}. \end{aligned} \tag{50}$$

A similar calculation can be performed for for I_3, but it should be apparent from inspection that the result will be the same as for I_2 except that there will no longer be the term $(-1)^{n_2 + n_3}$. Although calculating the infinite sum seems daunting, we found that, for the examples calculated in this paper, only a few moments in the power spectrum and a few tens of moments in the window function were necessary to get very good convergence, making the problem numerically tractable.

3.3.   Shot Noise

In the previous section we considered the variance in the multipole moments for a single measurement in the continuum limit. For magnitude limited surveys, the number of galaxies contained in a given volume will fall with redshift until the variance arising from finite sampling, *i.e.* shot noise, can become comparable and then dominate the cosmic variance. To accurately determine the sensitivity of real surveys, one must accurately model the shot noise as well. In this sub–section, we shall discuss the shot noise calculation, leaving the details to the appendix.

Recall in eq. (38) that to calculate the covariance $\sigma^2_{\ell,\ell'}$ we needed to find the ensemble average of a four point moment

$$\langle (N_i - \langle N_i \rangle)(N_j - \langle N_j \rangle)(N_k - \langle N_k \rangle)(N_l - \langle N_l \rangle) \rangle, \tag{51}$$

which was summed over all indices. We claimed that in the limit of large n this average could be reduced to eq. (39), and we will now show this explicitly while calculating all other terms. We can rewrite the above average to explicitly include terms in which one or more of the indices are equal. The result is

$$\langle (N_i - \langle N_i \rangle)(N_j - \langle N_j \rangle)(N_k - \langle N_k \rangle)(N_l - \langle N_l \rangle) \rangle + \tag{52}$$
$$+ \quad \left(\delta_{ij} \langle (N_i - \langle N_i \rangle)^2 (N_k - \langle N_k \rangle)(N_l - \langle N_l \rangle) \rangle + 5 \text{ permutations} \right)$$
$$+ \quad \left(\delta_{ij} \delta_{kl} \langle (N_i - \langle N_i \rangle)^2 (N_k - \langle N_k \rangle)^2 \rangle + 2 \text{ permutations} \right)$$
$$+ \quad \left(\delta_{ij} \delta_{jk} \langle (N_i - \langle N_i \rangle)^3 (N_l - \langle N_l \rangle) \rangle + 3 \text{ permutations} \right)$$
$$+ \quad \left(\delta_{ij} \delta_{jk} \delta_{kl} \langle (N_i - \langle N_i \rangle)^4 \rangle,$$

where different indices are presumed to be unequal. Since in the limit of infinitesimal volumes $\langle N_i^n \rangle = n \phi(\vec{s}_i) d^3 s_i$, the ensemble averages reduce to

$$n^4 \phi(\vec{s}_i) \phi(\vec{s}_j) \phi(\vec{s}_k) \phi(\vec{s}_l) d^3 s_i d^3 s_j d^3 s_k d^3 s_l \left[\xi_{ij} \xi_{kl} + \xi_{ik} \xi_{jl} + \xi_{il} \xi_{jk} \right] \tag{53}$$
$$+ \quad \left(\delta_{ij} n^3 \phi(\vec{s}_j) \phi(\vec{s}_k) \phi(\vec{s}_l) d^3 s_j d^3 s_k d^3 s_l \xi_{kl} + 5 \text{ permutations} \right)$$
$$+ \quad \left(\delta_{ij} \delta_{kl} n^2 \phi(\vec{s}_i) \phi(\vec{s}_k) d^3 s_i d^3 s_k (1 + \xi_{ik}) + 2 \text{ permutations} \right)$$
$$+ \quad \left(\delta_{ij} \delta_{jk} \phi(\vec{s}_k) \phi(\vec{s}_l) d^3 s_k d^3 s_l \xi_{kl} + 3 \text{ permutations} \right)$$
$$+ \quad \delta_{ij} \delta_{jk} \delta_{kl} n \phi(\vec{s}_k) d^3 s_i,$$

for Gaussian fields. The highest order term in n is the one which we have previously calculated, while the others are small in comparison when n is large. What remains is to substitute eq. (53) into eq. (38) and

evaluate all the terms. These details are left to the appendix, but here we give the results. To all orders in n, the shot noise contribution to the variance of the multipole moments of the power spectrum is

$$
\begin{aligned}
\sigma^2_{shot\ \ell,\ell'}(K,K') \ = \ & \sqrt{(2\ell+1)(2\ell'+1)}\frac{1}{N}\sum_{i,j,m_j}\sum_{n_1=|\ell-i|}^{\ell+i}\sum_{n_2=|\ell'-j|}^{\ell'+j}\left\{\left[2\left[(-1)^{n_2+m_j}(-1)^{j+m_j}\right]\right.\right. \\
& \times \ \frac{(4\pi)^{5/2}}{\sqrt{2i+1}(2n_1+1)(2n_2+1)(2j+1)}\tilde{w}^{(2)}_j(K,K') \\
& \times \ \left.\mathcal{S}_{n_1,n_2;i}C_{n_1,i,n_2;m_j}C_{n_1,\ell,i;-m_j}C_{n_2,\ell',n_2;m_j}\right] \\
& + \ \delta_{\ell 0}\delta_{\ell'0}\frac{4\pi}{N^2}\int\frac{k^2dk}{(2\pi)^3}\tilde{W}^{(2,2)}(k)P_0(k) + \delta_{\ell\ell'}\left[1+(-1)^\ell\right]\frac{1}{N^2}\tilde{W}^{(2,2)}(K,K') \\
& + \ \left[\left(\frac{4\pi}{N}\right)^2\frac{\bar{V}}{V_w}(2\ell+1)(2\ell'+1)\sum_n\int\frac{k^2dk}{(2\pi)^3}P_n(k)\int\frac{s^2ds}{V_w}G(s)\left(i^{\ell+\ell'+n}+i^{\ell-\ell'+n}\right)\right. \\
& \times \ \left.j_\ell(Ks)j_{\ell'}(K's)j_n(ks)\begin{pmatrix}\ell & \ell' & n \\ 0 & 0 & 0\end{pmatrix}^2\right] \\
& + \ \delta_{\ell 0}\frac{2}{N^2}\frac{4\pi}{2\ell'+1}\int\frac{k^2dk}{(2\pi)^3}\tilde{W}^{(3,1)}_{\ell'}(K',k)P_{\ell'}(k) \\
& + \ \delta_{\ell'0}\frac{2}{N^2}\frac{4\pi}{2\ell+1}\int\frac{k^2dk}{(2\pi)^3}\tilde{W}^{(3,1)}_{\ell}(K,k)P_{\ell}(k) \\
& + \ \left.\delta_{\ell 0}\delta_{\ell'0}\frac{1}{N^3}\int\frac{d^3s}{V_w}w^4(s)\right\},
\end{aligned}
\tag{54}
$$

where

$$
G(\vec{s}) \equiv \frac{1}{2}\int\frac{d^3s'}{V_w^2}w^2(\frac{\vec{s}+\vec{s'}}{2})w^2(\frac{\vec{s}-\vec{s'}}{2})
\tag{55}
$$

and $\tilde{w}^{(p)}(\vec{k})$, $\tilde{W}^{(p,q)}(\vec{k})$ and $\tilde{W}^{(p,q)}_\ell(\vec{k})$ are defined above. In calculating these terms, we have assumed that the window function is spherically symmetric (implying $G(\vec{s}) = G(s)$), and also that the selection function ϕ is constant over the window, leading us to define $N = n\phi V_w$.

Since factors of n now appear explicitly in the shot noise terms, we require an estimate of the number density of galaxies as a function of redshift. The SDSS is an r-band magnitude-limited survey, so we need an appropriate galaxy luminosity function. An analysis of the same band for the galaxies in the Las Campanas redshift survey performed by Lin $et.\ al.$ (1996) has shown that the galaxy luminosity function is well fit by a Schechter function with $M^* = -20.29 \pm 0.02 + 5\ \log h$, $\alpha = -0.70 \pm 0.05$, and $\phi^* = 0.019 \pm 0.001\ h^3$. For our models we will presume that these parameters will also well describe the SDSS and take $h = 0.7$, consistent with the fiducial models we shall consider. To get 10^6 galaxies in the total survey volume, we choose an apparent magnitude limit of $m = 17.6$ consistent with the limit $m \approx 18$ quoted for the SDSS. A second map has also been proposed for the SDSS data that would generate a volume limited survey of 10^5

bright red galaxies with an approximate depth of $z = 0.5$ (Szalay (1997)). We found in our calculations that the density of galaxies for this survey was insufficient to significantly strengthen the constraints derived from the proposed magnitude limited survey, so we shall only analyze the latter in detail.

4. Sensitivity of Redshift Surveys

The next generation of redshift surveys, particularly the Sloan Digital Sky Survey, will probe large fractions of the sky observing up to a million galaxy redshifts. In this section, we would like to estimate how well SDSS, the largest proposed survey, can measure cosmological parameters from redshift space distortions. Specifically, we want to determine whether the geometric and evolutionary effects which occur at larger redshifts can break the degeneracy between the matter density and the bias, producing a clean signal for Ω_0. To that end, we consider a pair of fiducial models which have good concordance with observation and perform a statistical fit of cosmological parameters.

In the last section, we calculated the mean and variance of the multipole moments of the power spectrum measured in small sub–volumes of a large survey. For nearby samples, when geometric effects are unimportant, the ratios of any two moments depend only on β and the shape of the power spectrum, but not its normalization. Cosmological models are roughly degenerate in parameter choices for which $\beta = \Omega_0^{0.6}/b_0$ is a constant. We shall reconsider these ratios with the inclusion of geometric and evolutionary effects to see if this degeneracy can be broken in deeper surveys. The ratio of $\bar{p}_2/\bar{p}_0$ is considered as it yields the largest signal to noise proportion; our numerical results show that the non–linear clustering effects tend to suppress the signal in higher moments. Two types of fiducial models with $\Omega_0 = 0.3$ are considered, one an open model with $\lambda = 0$ and the other a flat model with $\lambda = 0.7$, as these choices are favored by observations of large scale structure, *e.g.* Tadros & Efstathiou (1995). For the dark matter component, we assume Cold Dark Matter, with a fixed shape parameter of $\Gamma \equiv \Omega_0 h_0 = 0.2$, which we use for all test models so as to deconvolve the redshift distortion effects from the effects of the of the changing power spectrum shape. Furthermore, Γ should be well measured in the upcoming redshift surveys by direct calculation of the power spectrum, and thus should be a fixed input. To model the non–linear dispersion, we choose a velocity of 300 km/s which corresponds to a length scale of 3 h^{-1} Mpc in Hubble units.

In comparing our test models to the fiducial ones, we must consider how one can best extract measurements of $\bar{p}_2/\bar{p}_0$ from a real survey. Cole, Fisher, & Weinberg (1995) examined n-body simulations in the case of no evolution or geometric effects. They took repeated sub–samples by randomly locating the

center of a window function and measuring the multipole moments in these sub–volumes. To ensure that the data are statistically independent, the typical separation distance between sub–samples is chosen to be the size of the window, otherwise, the overlap will introduce correlations. We reconsider this procedure, only now, the effects of redshift evolution and geometry will be included to more accurately model real surveys.

Clearly, to maximize the number of measurements, one ought to consider the smallest possible sub–volume. Unfortunately, non–linear effects dominate on small scales and our simple model of the non–linear velocity dispersion will break down. In n-body simulations, Cole, Fisher, & Weinberg (1995) found that the corrected linear models failed at wavelengths below about $20h^{-1}$ Mpc, so we shall take this to be the lower limit on which we can apply our linear calculations. Thus we consider a Gaussian window—chosen because its multipole moments are expressible analytically—with a radius in redshift space which corresponds locally to $r_0 = 20h^{-1}$ Mpc, where $r = H_0^{-1}z$. For our calculations of $\bar{p}_\ell(K)$, we selected $K = 2\pi/20 \ h\mathrm{Mpc}^{-1}$ consistent with the n-body results. We then divided space into slices $\pi z^2 dz$ corresponding to the π steradians to be covered by the SDSS, with dz equal to the window diameter of $40h^{-1}$ Mpc locally. Redshifts corresponding to less than twice the window width were ignored because they were not sufficiently distant for the distant observer (*i.e.* small angle) approximation assumed in eq. (6) of §2 to be valid. High redshift data was dropped once it became shot-noise dominated and no longer contributed significantly to the fits. In each of these slices we divided the shell volume by the window volume to determine the number of statistically independent measures that were available at that redshift. To determine the expected variance of the $\bar{p}_\ell(K)$'s, we divided the variance for a single measurements by the number of volumes, the standard suppression for multiple independent measurements, and calculated the errors in their ratios using simple propagation of errors. In other words, the error bars were determined by

$$\sigma^2 = \frac{1}{\mathcal{N}} \frac{\bar{p}_2^2}{\bar{p}_0^2} \left(\frac{\sigma_{00}^2}{\bar{p}_0^2} - 2\frac{\sigma_{02}^2}{\bar{p}_0\bar{p}_2} + \frac{\sigma_{22}^2}{\bar{p}_2^2} \right), \tag{56}$$

where $\mathcal{N}$ is the number of independent volumes in the particular redshift slice. The result was a set of data points showing the expected mean and deviation for the redshift bins one would reasonably choose when analyzing a real survey. We then repeated this procedure for $r_0 = 40h^{-1}$ Mpc, the smallest scale which is independent of the $20h^{-1}$ Mpc data. We found that data on both scales were necessary to constrain all the model parameters, while including larger scales was not significantly more constraining.

To compare with other cosmologies, we calculated the mean values of $\bar{p}_2/\bar{p}_0$ for models with varying Ω_0, b_0, and σ and calculated a χ^2 using the fiducial model. We defined our confidence limit as the surface of

constant χ^2 into which a given percentage of best fit model parameters would fall for multiple realizations of the fiducial ensemble. For example, the 95% confidence limit is defined such that the best fit model parameters will fall within that surface 95% of the time. Since we have three fitting parameters, there are three degrees of freedom in our χ^2 statistics.

5.   Results and Conclusions

In figure 1, we show two examples of $\bar{p}_2/\bar{p}_0$ plotted as a function of redshift, where we have removed the shot noise contributions to the mean values. In the upper panel, we show a fiducial model (stars) of $\Omega_0 = 0.3$, $\lambda_0 = 0.7$ and $b_0 = 1.0$ with 1σ error bars, compared to a test model (boxes) of $\Omega_0 = 0.48$, $\lambda_0 = 0.52$, and $b_0 = 1.2$; and in the lower panel, we show a fiducial model of $\Omega_0 = 0.3$, $\lambda_0 = 0$, and $b_0 = 1.0$ compared to a test model of $\Omega_0 = 0.85$, $\lambda_0 = 0$, and $b_0 = 1.60$. In each example, data for both the $20h^{-1}$ Mpc and $40h^{-1}$ Mpc windows are shown, where the larger window produces higher values for the ratio, and all models use the fiducial value for σ. Data was terminated at the redshift after which shot noise dominated the result. Both comparison models are barely accepted at the 95% confidence limit when the fiducial velocity dispersion was used for each. We emphasize this with figure 2 which shows the 68%, 95%, and 99% confidence limits for flat, cosmological constant test models given a fiducial $\Omega_0 = 0.3$, $\lambda_0 = 0.7$, and $b_0 = 1.0$ model like that of figure 1. These surfaces are projections of the full three dimensional confidence volume, and they represent the confidence limits when the non–linear velocity dispersion is unconstrained by other data. The dotted line shows the curve $\Omega_0^{0.6}/b_0 = 0.49$ which is the degeneracy one expects locally when redshift effects are ignored. We do not show a figure comparing open models to our fiducial cosmological constant model, as none were acceptable at even the 99% confidence limit. In figure 3 we switch to an open $\Omega_0 = 0.3$, $\lambda_0 = 0$, and $b_0 = 1.0$ fiducial model and test it against other open models. Finally, in figure 4 we test this fiducial model against flat cosmological constant models.

In figure 1 we see the significant redshift dependence of $\bar{p}_2/\bar{p}_0$, where we would expect none in models which ignore geometry and evolution. Naively fitting a horizontal line representing models with no redshift dependent effects, we get a best fit $\beta_0 = 0.45$ for our fiducial flat model and $\beta_0 = 0.26$ for our fiducial open model using only the $20h^{-1}$ Mpc data; we expect a value of $\beta_0 \approx 0.5$ in both cases. Thus we verify that there will be systematic errors in models which ignore redshift evolution effects, a point emphasized by Nakamura, Matsubara, & Suto (1997). The slopes of theses data can be qualitatively understood by considering the competing effects of the evolution of β and geometry. At deeper redshifts, $\Omega(z) \rightarrow 1$ and β increases,

tending to increase the value of the ratio of $\bar{p}_2/\bar{p}_0$ for the power spectrum and non–linear dispersion we have chosen. Geometry, on the other hand, pushes the effective scale which is probed to smaller scales and thus larger values of the wave number, *i.e.* objects appear more spread out at high redshift. Non–linear dispersion tends to drive the multipole ratio to lower values as the effective K grows, so geometric effects tend to decrease the multipole ratios. Thus we have two competing effects which determine the slope of the multipole ratio curve as a function of redshift. For $\Omega_0 = 1$ models, β remains fairly constant—changing only due to the evolution in bias. These models are dominated by geometric effects and thus have the steepest negative slope. The high cosmological constant model has the flattest slope, because the rapid evolution in β just cancels the geometric effects, while the open models fall in between. The effects of changing the non–linear velocity dispersion generally shifts the overall normalization of the data without affecting the slope significantly. With only the $20h^{-1}$ Mpc data, there is a degeneracy between b_0 and σ. To break it, we need to include the data from the larger window, as the change in normalization is different for the two window scales. We also note that the nearly flat evolution of the multipole ratio also explains why open models cannot fit our fiducial cosmological model; all are too sloped to be good fits to the data.

Regarding the determination of cosmological parameters, the results shown in the last three figures reveal interesting results. For the fiducial open model (figure 3), the degeneracy between models of differing Ω_0 and b_0 is weakly broken, and the redshift distortions do permit us to distinguish between the open fiducial model and $\Omega_0 = 1$ models at the 95% limit, although not at the 99% limit. However, when we tried a fiducial $\Omega_0 = 0.4$ model, $\Omega_0 = 1$ was allowed at the 95% limit, so one can conclude that an open universe with $\Omega_0 = 0.3$ is just on the edge of being able to exclude critical matter density models. For the case of the flat fiducial model (figure 2), we see that values of $\Omega_0 > 0.48$ are ruled out at the 95% confidence limit, producing significantly stronger constraints than is the case for the open model. In figure 4 we test the likelihood of confusing flat models with open. We see that models with some cosmological constant can be confused with the open fiducial model. Overall, we conclude that redshift surveys like SDSS may just be able to determine bias and evolution independent measures of cosmological parameters, at least in discriminating the extremes of $\Omega_0 = 1$ and $\Omega_0 = 0.3$. although only nominally so for open universes. We also see that naive estimates of β which ignore redshift evolution are systematically biased towards smaller values.

When it comes to determining cosmological parameters, the ideal result is to measure them with several independent observations, looking for a consistency of result which would demonstrate that we understand the fundamentals of cosmology or an inconsistency that would indicate a failure of some aspect

of current theory. It has been suggested that redshift surveys offer promise of being a reliable source of data for such cosmological parameter determination, when appropriate statistics are applied. Simply measuring the change in the mean number of galaxies as a function of redshift is inadequate, because we cannot observe the gravitational component in the absence of reliable models for the non–gravitational evolution. Alternatively, one can consider the redshift evolution of the multipole moments of the linear power spectrum, which are driven only by gravity, and hope to cleanly observe cosmological effects. Our results suggest that if the true model of the universe contains a large cosmological constant, then Ω_0 is tightly constrained; however, for open models the the extremes of $\Omega_0 = 1$ and $\Omega_0 = 0.3$ only can marginally be distinguished (95% but not 99% confidence limit) in surveys on the scale of SDSS.

REFERENCES

Alcock, C. & Paczyński, B. 1979, Nature, 281, 358

Ballinger, W. E. Peacock, J. A. & Heavens, A. F. 1996, preprint, astro-ph/9605017

Cole, S., Fisher, K. B., & Weinberg, D. H. 1995, MNRAS, 267, 785

Cole, S., Fisher, K. B., & Weinberg, D. H. 1995, MNRAS, 275, 515

A. R. Edwards, Angular Momentum in Quantum Mechanics, (Princeton: Princeton University Press)

Fry, J. 1996, ApJ, 461, L5

Gunn, J. E., & Weinberg, D. H., in "Wide Field Spectroscopy and the Distant Universe", ed. S. J. Maddox & A. Aragón–Salamanca (Singapore: World Scientific); Strauss, M. A. 1997, preprint astro-ph/9610032

Kaiser, N. 1987, MNRAS, 227,1

Lin, H. *et. al.* 1996, ApJ, 464, 60

Loveday, J., Efstathiou, G., Maddox, S. J., Peterson, B. A. 1996, ApJ, 486, 1L

Matsubara, T. & Suto, Y. 1996, ApJ, 470, L1

Nakamura, T. T., Matsubara, T., & Suto, Y. 1997, preprint, astro-ph/9706034

Peebles, P. J. E. 1980, The Large Scale Structure on the Universe (Princeton: Princeton University Press)

Schulten, K., Gordon, G. 1975, J. of Math. Phys. 10, 1961

Szalay, A., personal communication

Tadros, H., & Efstathiou, G. 1995, MNRAS, 276, L45

Appendix A.

In §3.3, we discussed the complete expansion of the variance including the lower order terms in n. Here we shall show the details of calculating the shot noise terms ignored in §3.2. We begin with the leading order correction in eq. (52), which when substituted into eq. (38), may be written

$$\frac{2\ell+1}{4\pi}\frac{2\ell'+1}{4\pi}\int d\Omega_K d\Omega_{K'}\mathcal{P}(\mu_K)\mathcal{P}(\mu_{K'})\frac{1}{n^4 V_w^4} \tag{1}$$

$$\times \sum_{i,j,k,l}\frac{w(\vec{s}_i)w(\vec{s}_j)w(\vec{s}_k)w(\vec{s}_l)}{\phi(\vec{s}_i)\phi(\vec{s}_j)\phi(\vec{s}_k)\phi(\vec{s}_l)}e^{i\vec{K}\cdot(\vec{s}_i-\vec{s}_j)}e^{i\vec{K}'\cdot(\vec{s}_k-\vec{s}_l)}$$

$$\times \left[\delta_{ij}\langle(N_i-\langle N_i\rangle)^2(N_k-\langle N_k\rangle)(N_l-\langle N_l\rangle)\rangle + \delta_{ik}\langle(N_i-\langle N_i\rangle)^2(N_j-\langle N_j\rangle)(N_l-\langle N_l\rangle)\rangle\right.$$

$$+\ \delta_{il}\langle(N_i-\langle N_i\rangle)^2(N_j-\langle N_j\rangle)(N_k-\langle N_k\rangle)\rangle + \delta_{jk}\langle(N_i-\langle N_i\rangle)(N_j-\langle N_j\rangle)^2(N_l-\langle N_l\rangle)\rangle$$

$$+\ \delta_{jl}\langle(N_i-\langle N_i\rangle)(N_j-\langle N_j\rangle)^2(N_k-\langle N_k\rangle)\rangle + \delta_{kl}\langle(N_i-\langle N_i\rangle)(N_j-\langle N_j\rangle)(N_l-\langle N_l\rangle)^2\rangle\left.\right].$$

Now let us digest this term by term. The first we shall designate $I_{1,1}$ and, recalling eq. (53), we may write it as

$$I_{1,1} = \frac{2\ell+1}{4\pi}\frac{2\ell'+1}{4\pi}\int d\Omega_K d\Omega_{K'}\mathcal{P}_\ell(\mu_K)\mathcal{P}_{\ell'}(\mu_{K'})\frac{1}{nV_w^4}\sum_{j,k,l}\frac{w^2(\vec{s}_j)}{\phi(\vec{s}_j)}w(\vec{s}_k)w(\vec{s}_l) \tag{2}$$

$$\times\ \xi_{k,l}e^{i\vec{K}'\cdot(\vec{s}_k-\vec{s}_l)}d^3s_j d^3s_k d^3s_l.$$

To allow us to evaluate many of the integrals that we will encounter in this section, we need to make a simplifying assumption about the selection function ϕ: that it is approximately constant in the window of interest. For small windows this should be reasonable, so we thus define $N = n\phi V_w$ to be the number of galaxies in a given window. Substituting in the Fourier expansion for the correlation function and evaluating the spatial integrals, it is straight forward to show

$$I_{1,1} = \delta_{l0}\frac{1}{N}\frac{\bar{V}}{V_w}\int d\Omega_{K'}\mathcal{P}_{\ell'}(\mu_{K'})\int\frac{d^3k}{(2\pi)^3}\tilde{W}^{(1,1)}(\vec{K}+\vec{k}')P(\vec{k}), \tag{3}$$

with

$$\bar{V} = \int d^3s\ w^2(\vec{s}). \tag{4}$$

and $W^{(p,q)}$ defined in eq. (33). Expanding everything into Legendre series and evaluating the angular integrals, one can show directly that

$$I_{1,1} = \delta_{l0}\frac{4\pi}{2\ell'+1}\frac{\bar{V}}{V_w}\int\frac{k^2 dk}{(2\pi)^3}P_{\ell'}(k)\tilde{W}_{\ell'}^{(1,1)}(K',k). \tag{5}$$

By inspection of eq. (1), one can also see that $I_{1,6}$ is equal to $I_{1,1}$ under the interchange of ℓ and ℓ'. However, looking at eq. (37), we see that both of these terms will cancel with the terms coming from $\langle \bar{p}_\ell \rangle \langle \bar{p}_{\ell'} \rangle$, so they are dropped below.

Now we continue on to consider the second term $I_{1,2}$ which may be written

$$
\begin{aligned}
I_{1,2} &= \frac{2\ell+1}{4\pi}\frac{2\ell'+1}{4\pi}\int d\Omega_K d\Omega_{K'} \mathcal{P}_\ell(\mu_K)\mathcal{P}_{\ell'}(\mu_{K'})\frac{1}{nV_w^4}\sum_{j,k,l}\frac{w^2(\vec{s}_k)}{\phi(\vec{s}_k)}w(\vec{s}_j)w(\vec{s}_l) \\
&\times \ \xi_{j,l}e^{i\vec{K}\cdot(\vec{s}_k-\vec{s}_j)}e^{i\vec{K}'\cdot(\vec{s}_k-\vec{s}_l)}d^3s_j d^3s_k d^3s_l.
\end{aligned}
\tag{6}
$$

Performing the spatial integrals, we find

$$
\begin{aligned}
I_{1,2} &= \frac{1}{N}\frac{2\ell+1}{4\pi}\frac{2\ell'+1}{4\pi}\int d\Omega_K d\Omega_{K'}\mathcal{P}_\ell(\mu_K)\mathcal{P}_{\ell'}(\mu_{K'}) \\
&\quad \int \frac{d^3k}{(2\pi)^3}\tilde{w}^{(2)}(\vec{K}+\vec{K}')\tilde{w}(\vec{k}+\vec{K})\tilde{w}(\vec{k}-\vec{K}')P(\vec{k}).
\end{aligned}
\tag{7}
$$

with $\tilde{w}^{(2)}$ defined in eq. (31). Let us take a closer look at the integral

$$
\int \frac{d^3k}{(2\pi)^3}P(\vec{k})\tilde{w}(\vec{k}+\vec{K})\tilde{w}(\vec{k}-\vec{K}').
\tag{8}
$$

If we expand each term into a Legendre series, and then expand those Legendre polynomials into spherical harmonics using the addition theorem, we can see after evaluating the angular integrals that this term may be reduced to

$$
\sum_{n_1,n_2}\sum_{i=|n_1-n_2|}^{n_1+n_2}\sum_{m_1=-n_1}^{n_1}\frac{(4\pi)^2}{(2n_1+1)(2n_2+1)}\mathcal{S}_{n_1,n_2;i}(-1)^{n_2}\sqrt{\frac{4\pi}{2i+1}}Y_{n_1,-m_1}(\Omega_K)Y_{n_2,m_1}(\Omega_{K'})C_{n_1,i,n_2;m_1}
\tag{9}
$$

with $\mathcal{S}$ defined in eq. (48). To make further progress, we substitute the above back into eq. (8) and expand $\tilde{w}^{(2)}(\vec{K}+\vec{K}')$ in a spherical harmonic series. The rest of the work consists in evaluating various angular integral like we have already seen, so we shall just quote the final result:

$$
\begin{aligned}
I_{1,2} &= \sqrt{(2\ell+1)(2\ell'+1)}\frac{1}{N}\sum_{i,j,m_j}\sum_{n_1=|\ell-i|}^{\ell+i}\sum_{n_2=|\ell'-j|}^{\ell'+j}(-1)^{n_2+m_j} \\
&\quad \times \frac{(4\pi)^{5/2}}{\sqrt{2i+1}(2n_1+1)(2n_2+1)(2j+1)} \\
&\quad \times \tilde{w}_j^{(2)}(K,K')\mathcal{S}_{n_1,n_2;i}C_{n_1,i,n_2;m_j}C_{n_1,\ell,i;-m_j}C_{n_2,\ell',n_2;m_j}.
\end{aligned}
\tag{10}
$$

By inspection one can see that the term $I_{1,5}$ is equal to $I_{1,1}$. The remaining terms $I_{1,3}$ and $I_{1,4}$ are equivalent to $I_{1,2}$ under the transformation $-1^{n_2}\rightarrow -1^j$.

The next correction term in eq. (52) has the form:

$$\frac{2\ell+1}{4\pi}\frac{2\ell'+1}{4\pi}\int d\Omega_K d\Omega_{K'}\mathcal{P}(\mu_K)\mathcal{P}(\mu_{K'})\frac{1}{n^4 V_w^4} \tag{11}$$

$$\times \quad \sum_{i,j,k,l}\frac{w(\vec{s}_i)w(\vec{s}_j)w(\vec{s}_k)w(\vec{s}_l)}{\phi(\vec{s}_i)\phi(\vec{s}_j)\phi(\vec{s}_k)\phi(\vec{s}_l)}e^{i\vec{K}\cdot(\vec{s}_i-\vec{s}_j)}e^{i\vec{K}'\cdot(\vec{s}_k-\vec{s}_l)}$$

$$\times \quad [\delta_{ij}\delta_{kl}\langle N_j N_l\rangle + \delta_{ik}\delta_{jl}\langle N_k N_l\rangle + \delta_{il}\delta_{jk}\langle N_l N_k\rangle].$$

We refer to each of the terms here as $I_{2,1}$, $I_{2,2}$ and $I_{2,3}$ respectively. The first, $I_{2,1}$ can be expanded into the following:

$$I_{2,1} \quad = \quad \delta_{\ell 0}\delta_{\ell'0}\frac{1}{N^2}\left[\left(\frac{\bar{V}}{V_w}\right)^2 + \int \frac{d^3k}{(2\pi)^3}\tilde{W}^{1,1}P(\vec{k})\right], \tag{12}$$

after we insert the Fourier expansion for the correlation function and evaluate the spatial integrals. The first term will cancel with pieces from $\langle \bar{p}_\ell\rangle\langle\bar{p}_{\ell'}\rangle$, leaving the second. The angular part of the k integral is easily evaluated, leaving

$$I_{2,1} = \delta_{\ell 0}\delta_{\ell'0}\frac{4\pi}{N^2}\int \frac{k^2 dk}{(2\pi)^3}\tilde{W}^{(1,1)}(k)P_0(k). \tag{13}$$

For the second term in eq. (11), we shall not automatically evaluate the spatial integral because the results are simpler to reduce for some terms if we evaluate the angular parts first. Thus we write

$$I_{2,2} \quad = \quad \frac{1}{N^2}\frac{2\ell+1}{4\pi}\frac{2\ell'+1}{4\pi}\int d\Omega_K d\Omega'_K \mathcal{P}_l(\mu_K)\mathcal{P}'_l(\mu'_K)\sum_{k,l}w^2(\vec{s}_k)w^2(\vec{s}_l) \tag{14}$$

$$\times \quad e^{i\vec{s}_k\cdot(\vec{K}+\vec{K}')}e^{-i\vec{s}_l\cdot(\vec{K}+\vec{K}')}\left[1 + \int \frac{d^3k}{(2\pi)^3}e^{i\vec{k}\cdot(\vec{s}_k-\vec{s}_l)}P(\vec{k})\right]d^3 s_k d^3 s_l.$$

The first term, designated $I_{2,2A}$ evaluates to

$$I_{2,2A} = \frac{1}{N^2}\frac{2\ell+1}{4\pi}\frac{2\ell'+1}{4\pi}\int d\Omega_K d\Omega_{K'}\mathcal{P}_\ell(\mu_K)\mathcal{P}_{\ell'}(\mu_{K'})\tilde{W}^{(1,1)}(\vec{K}+\vec{K}'), \tag{15}$$

which, after expanding the widow function into a Legendre series and evaluating the angular integrals, reduces to

$$I_{2,2A} = \delta_{\ell\ell'}\frac{1}{N^2}\tilde{W}_\ell^{(1,1)}(K,K'). \tag{16}$$

To tackle the second term in eq. (14), we first define a new set of spatial variable $\vec{s} = \vec{s}_k - \vec{s}_l$ and $\vec{s}' = \vec{s}_k - \vec{s}_l$. We rewrite the spatial sums as integrals in the new variables, producing

$$I_{2,2B} \quad = \quad \frac{1}{N^2}\frac{2\ell+1}{4\pi}\frac{2\ell'+1}{4\pi}\int d\Omega_K d\Omega_{K'}\mathcal{P}_\ell(\mu_K)\mathcal{P}_{\ell'}(\mu_{K'}) \tag{17}$$

$$\times \quad \frac{1}{2}\int \frac{d^3 s\, d^3 s'}{V_w^2}w^2(\frac{\vec{s}+\vec{s}'}{2})w^2(\frac{\vec{s}-\vec{s}'}{2})e^{iK\cdot\vec{s}}e^{iK'\cdot\vec{s}}\int \frac{d^3 k}{(2\pi)^3}e^{i\vec{k}\cdot\vec{s}}P(\vec{k}).$$

The quantity

$$G(\vec{s}) \equiv \frac{1}{2} \int \frac{d^3 s \, d^3 s'}{V_w^2} w^2(\frac{\vec{s} + \vec{s}'}{2}) w^2(\frac{\vec{s} - \vec{s}'}{2}) \tag{18}$$

is a function only of the magnitude of $\vec{s}$ if w is spherically symmetric. The exponential has the following expansion in Legendre polynomials

$$e^{iK \cdot \vec{s}} = \sum i^n (2n + 1) j_n(Ks) \mathcal{P}_n(\mu_{K,s}), \tag{19}$$

which we may use, along with eq. (36) and eq. (45) to evaluate the angular integrals. The final result can be written

$$
\begin{aligned}
I_{2,2B} &= \left(\frac{4\pi}{N}\right)^2 (2\ell + 1)(2\ell' + 1) \sum_n \int \frac{k^2 dk}{(2\pi)^3} P_n(k) \int \frac{s^2 ds}{V_w} G(s) \\
&\times \begin{pmatrix} \ell & \ell' & n \\ 0 & 0 & 0 \end{pmatrix}^2 i^{\ell + \ell' + n} j_\ell(Ks) j_{\ell'}(K's) j_n(ks).
\end{aligned}
\tag{20}
$$

Inspection of the third term in eq. (14) reveals that $I_{2,3A} = (-1)^\ell I_{2,2A}$ while $I_{2,3B} = i^{-2\ell'} I_{2,2B}$.

Moving along, we see that the fourth term in eq. (52) is

$$
\begin{aligned}
&\frac{2\ell + 1}{4\pi} \frac{2\ell' + 1}{4\pi} \int d\Omega_K d\Omega_{K'} \mathcal{P}(\mu_K) \mathcal{P}(\mu_{K'}) \frac{1}{n^4 V_w^4} \\
&\times \sum_{i,j,k,l} \frac{w(\vec{s}_i) w(\vec{s}_j) w(\vec{s}_k) w(\vec{s}_l)}{\phi(\vec{s}_i) \phi(\vec{s}_j) \phi(\vec{s}_k) \phi(\vec{s}_l)} e^{i\vec{K} \cdot (\vec{s}_i - \vec{s}_j)} e^{i\vec{K}' \cdot (\vec{s}_k - \vec{s}_l)} \\
&\times [\delta_{ij}\delta_{jk}(\langle N_i N_l \rangle - \langle N_i \rangle \langle N_l \rangle) + \delta_{ij}\delta_{jl}(\langle N_i N_k \rangle - \langle N_i \rangle \langle N_k \rangle) \\
&+ \delta_{ik}\delta_{kl}(\langle N_i N_j \rangle - \langle N_i \rangle \langle N_j \rangle) + \delta_{jk}\delta_{kl}(\langle N_i N_j \rangle - \langle N_i \rangle \langle N_j \rangle)].
\end{aligned}
\tag{21}
$$

Defining $\tilde{W}_\ell^{(3,1)}$ according to eq. (35), it is straightforward to show that

$$I_{3,1} = \delta_{\ell 0} \frac{1}{N^2} \frac{4\pi}{2\ell + 1} \int \frac{k^2 dk}{(2\pi)^3} \tilde{W}_{\ell'}^{(3,1)}(K', k) P_{\ell'}(k). \tag{22}$$

The term $I_{3,2} = I_{3,1}$ while the terms $I_{3,3}$ and $I_{3,4}$ are equivalent to $I_{3,1}$ under the transformation $\ell \leftrightarrow \ell'$ and $K \leftrightarrow K'$. The final remaining term in eq. (38) can be evaluated trivially, producing

$$I_4 = \frac{1}{N^3} \delta_{\ell 0} \delta_{\ell' 0} \int \frac{d^3 s}{V_w} w^4(\vec{s}). \tag{23}$$

Putting all of these (eq. (5,10,13,16 ,20,22,23) yields eq. (54) .

Figure Captions

Fig. 1.— Comparing the redshift evolution of $\bar{p}_2/\bar{p}_0$ for different cosmological models. In the upper panel, the starred data points show our fiducial flat model ($\lambda_0 = 0.7, b_0 = 1$) with error estimates compared with the square data points which represent a $\lambda_0 = 0.48$, $b_0 = 1.2$ model. Data for windows of $20h^{-1}$ Mpc (lower curves) and $40h^{-1}$ Mpc (upper curves) are shown. The comparison model has the smallest value of λ_0 which is still acceptable at the 95% confidence limit. In the lower panel, the starred data represents our fiducial open model ($\Omega_0 = 0.3$) compared with an $\Omega_0 = 0.85$, $b_0 = 1.6$ model, which is again just acceptable at the 95% confidence limit.

Fig. 2.— The 68%, 95%, and 99% confidence limits for Ω_0 and b_0 with σ unconstrained, when comparing flat, cosmological constant models with a fiducial $\Omega_0 = 0.3$, $\lambda_0 = 0.7$, $\sigma = 300$ km/s, and $b_0 = 1$ model. The dashed curve plots $\Omega_0^{0.6}/b_0 = .5$, the naive degeneracy expected

Fig. 3.— The 68%, 95%, and 99% confidence limits for Ω_0 and b_0 with σ unconstrained, when comparing open models with a fiducial $\Omega_0 = 0.3$, $\lambda_0 = 0$, $\sigma = 300$ km/s, and $b_0 = 1$ model. The dashed curve plots $\Omega_0^{0.6}/b_0 = .5$, the naive degeneracy expected

Fig. 4.— The 68%, 95%, and 99% confidence limits for Ω_0 and b_0 with σ unconstrained, when comparing flat, cosmological constant models with a fiducial $\Omega_0 = 0.3$, $\lambda_0 = 0$, $\sigma = 300$ km/s, and $b_0 = 1$ model. The dashed curve plots $\Omega_0^{0.6}/b_0 = .5$, the naive degeneracy expected